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SPENDING CONSIDERATION FOR DISSIMILAR COLD STANDBY UNITS WITH RANDOM CHECK

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ABSTRACT;
This paper deals with the cost-benefit analysis of a two-unit cold standby system. The author has considered a cold standby system with two dissimilar units. These two units are named as priority unit (P-unit) and standby unit (S-unit). The main working unit is P-unit but on failure of this unit we may online S-unit through an imperfect switching device. This S-unit is not efficient to fulfill the requirements similar to P-unit. In this study, the author has been used a random check for S-unit during its non-operation period.

KEY WORDS: cold standby system, efficient

INTRODUCTION:
Such type of system can be seen in daily life. For example, we may take air conditioner as P-unit and cooler as S-unit. On failure of air conditioner we may use cooler to maintain room temperature. Cooler is not efficient as compared to air conditioner to keep the room cool and dry. On failure of air conditioner we may use cooler but it is possible that at the time of need we find that cooler is not working due to non-operation for a long period. Therefore, it is essential to check randomly the cooler during operable condition of air conditioner. Since the system under consideration is Non-Markovian, the author has used supplementary variables to convert this in Markovian. Transition-state diagram of the system has been shown in fig-1. Table-1 gives details of various system states. Mathematical model of the system has been solved with the help of Laplace transform. Asymptotic behavior of system and some particular cases have been computed to improve practical utility of the model. Availability function and cost function of considered system have been obtained. At the last, we appended a numerical illustration together with its graphical representation to highlight important results of this study.

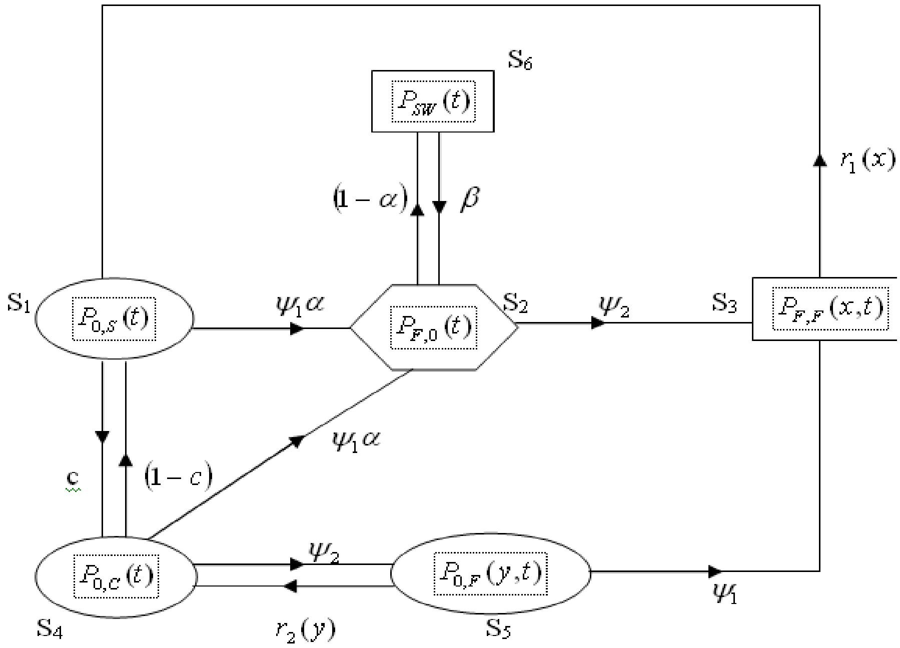

Fig-1: Transition-state diagram

Table-1: State description

S.N.	State	Probability	P-Unit	S-Unit	Switching Device	System state
1	S_1	$P_{0,S}$	Operable	Standby	off	Operable
2	S_2	$P_{F,0}$	Failed	Operable	on	Degraded
3	S_3	$P_{F,F}$	Failed	Failed	on	Failed
4	S_4	$P_{0,C}$	Operable	Random Check	off	Operable
5	S_5	$P_{0,F}$	Operable	Failed	off	Operable
6	S_6	P_{SW}	Failed	Operable	Failed	Failed

FORMULATION OF MATHEMATICAL MODEL:

Using probability consideration and limiting procedure, we obtain the following set of difference-differential equations, governing the behaviour of considered system, continuous in time and discrete in space:

$$\left[\frac{d}{dt}+c+\psi_1\alpha\right]P_{0,S}(t)=(1-c)P_{0,C}(t)+\int_0^\infty P_{F,F}(x,t)r_1(x)dx \qquad \ldots(1)$$

$$\left[\frac{d}{dt}+\psi_2+(1-\alpha)\right]P_{F,0}(t)=\beta\, P_{SW}(t)+\psi_1\alpha\left[P_{0,C}(t)+P_{0,S}(t)\right] \qquad \ldots(2)$$

$$\left[\frac{d}{dt}+\beta\right]P_{SW}(t)=(1-\alpha)\,P_{F,0}(t) \qquad \ldots(3)$$

$$\left[\frac{d}{dt}+\psi_1\alpha+\psi_2+(1-c)\right]P_{0,C}(t)=c\,P_{0,S}(t)+\int_0^\infty P_{0,F}(y,t)r_2(y)dy \qquad \ldots(4)$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+r_1(x)\right]P_{F,F}(x,t)=0 \qquad \ldots(5)$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\psi_1+r_2(y)\right]P_{0,F}(y,t)=0 \qquad \ldots(6)$$

BOUNDARY CONDITIONS ARE

$$P_{F,F}(0,t)=\psi_2\,P_{F.0}(t)+\psi_1 P_{0,F}(t) \qquad \ldots(7)$$

$$P_{0,F}(0,t)=\psi_2\,P_{0,C}(t) \qquad \ldots(8)$$

INITIAL CONDITIONS ARE

$P_{0,S}(0)=1,$ otherwise zero.

SOLUTION OF THE MODEL :

Taking Laplace transforms of equations (1) through (8) subjected to initial conditions (9), we obtain:

$$[s+c+\psi_1\alpha]\overline{P}_{0,S}(s)=1+(1-c)\overline{P}_{0,C}(s)+\int_0^\infty \overline{P}_{F,F}(x,s)r_1(x)dx \quad \ldots(10)$$

$$[s+\psi_2+(1-\alpha)]\overline{P}_{F,0}(s)=\beta\,\overline{P}_{SW}(s)+\psi_1\alpha[\overline{P}_{0,C}(s)+\overline{P}_{0,S}(s)] \quad \ldots(11)$$

$$[s+\beta]\overline{P}_{SW}(s)=(1-\alpha)\,\overline{P}_{F,0}(s) \quad \ldots(12)$$

$$[s+\psi_1\alpha+\psi_2+(1-c)]\overline{P}_{0,C}(s)=c\,\overline{P}_{0,S}(s)+\int_0^\infty \overline{P}_{0,F}(y,s)r_2(y)dy \quad \ldots(13)$$

$$\left[\frac{\partial}{\partial x}+s+r_1(x)\right]\overline{P}_{F,F}(x,s)=0 \quad \ldots(14)$$

$$\left[\frac{\partial}{\partial y}+s+\psi_1+r_2(y)\right]\overline{P}_{0,F}(y,s)=0 \quad \ldots(15)$$

$$\overline{P}_{F,F}(0,s)=\psi_2\,\overline{P}_{F.0}(s)+\psi_1\overline{P}_{0,F}(s) \quad \ldots(16)$$

$$\overline{P}_{0,F}(0,s)=\psi_2\,\overline{P}_{0,C}(s) \quad \ldots(17)$$

Integrating equation (14) with the help of boundary condition (16), we get

$$\overline{P}_{F,F}(x,s)=[\psi_2\overline{P}_{F.0}(s)+\psi_1\overline{P}_{0,F}(s)]\exp\left\{-sx-\int r_1(x)dx\right\}$$

integrating this again w.r.t. 'x' from 0 to ∞, we have

$$\overline{P}_{F,F}(s)=[\psi_2\overline{P}_{F.0}(s)+\psi_1\overline{P}_{0,F}(s)]\frac{1-\overline{S}_1(s)}{s}$$

or, $\overline{P}_{F,F}(s)=[\psi_2\overline{P}_{F.0}(s)+\psi_1\overline{P}_{0,F}(s)]D_1(s)$ …(18)

Similarly, solving (15) subjected to (17), we obtain

$$\overline{P}_{0,F}(y,s)=\psi_2\overline{P}_{0,C}(s)\exp\left\{-(s+\psi_1)y-\int r_2(y)dy\right\}$$

$$\Rightarrow \overline{P}_{0,F}(s)=\psi_2\overline{P}_{0,C}(s)D_1(s+\psi_1) \quad \ldots(19)$$

Simplifying (13) with the help of (19), we have

$$[s+\psi_1\alpha+\psi_2+(1-c)]\overline{P}_{0,C}(s)=c\overline{P}_{0,S}(s)+\psi_2\overline{P}_{0,C}(s)\overline{S}_2(s+\psi_1)$$

$$\Rightarrow \overline{P}_{0,C}(s)=\frac{c\overline{P}_{0,S}(s)}{s+\psi_1\alpha+\psi_2\{1-\overline{S}_2(s+\psi_1)\}+(1-c)}$$

or, $\overline{P}_{0,C}(s)=A(s)\overline{P}_{0,S}(s)$ …(20)

Simplifying (12), we obtain

$$\overline{P}_{SW}(s)=\frac{(1-\alpha)\overline{P}_{F,0}(s)}{(s+\beta)} \quad \ldots(21)$$

Now again, simplifying (11) with the help of related relations

$$[s+\psi_2+(1-\alpha)]\overline{P}_{F,0}(s)=\frac{\beta(1-\alpha)}{(s+\beta)}\overline{P}_{F,0}(s)+\psi_1\alpha\overline{P}_{0,S}(s)[1+A(s)]$$

$$\Rightarrow \overline{P}_{F,0}(s)=\frac{\psi_1\alpha[1+A(s)]\overline{P}_{0,S}(s)}{s+\psi_2+\frac{s(1-\alpha)}{s+\beta}}$$

or, $\overline{P}_{F,0}(s)=B(s)\overline{P}_{0,S}(s)$ …(22)

Finally simplifying (10) subjected to relevant expressions,

$$[s+c+\psi_1\alpha]\overline{P}_{0,S}(s)=1+(1-c)A(s)\overline{P}_{0,S}(s)+\psi_2[B(s)+\psi_1A(s)D_2(s+\psi_1)]\overline{P}_{0,S}(s)\overline{S}_1(s)$$

$$\Rightarrow \overline{P}_{0,S}(s)=\frac{1}{E(s)}$$

Thus, we obtain the l.t. Of sate probabilities of fig-1 in terms of e(s) as follows:

$$\overline{P}_{0,S}(s) = \frac{1}{E(s)} \quad \ldots(23)$$

$$\overline{P}_{F,0}(s) = \frac{B(s)}{E(s)} \quad \ldots(24)$$

$$\overline{P}_{SW}(s) = \frac{(1-\alpha)B(s)}{(s+\beta)E(s)} \quad \ldots(25)$$

$$\overline{P}_{0,C}(s) = \frac{A(s)}{E(s)} \quad \ldots(26)$$

$$\overline{P}_{F,F}(s) = \frac{\psi_2 B(s)}{E(s)} D_1(s) \quad \ldots(27)$$

$$\overline{P}_{0,F}(s) = \frac{\psi_2 A(s)}{E(s)} D_2(s+\psi_1) \quad \ldots(28)$$

where, $A(s) = \dfrac{c}{s+\psi_1\alpha+\psi_2\left\{1-\overline{S}_2(s+\psi_1)\right\}+(1-c)}$ …(29)

$$B(s) = \frac{\psi_1\alpha[1+A(s)]}{s+\psi_2+\dfrac{s(1-\alpha)}{s+\beta}} \quad \ldots(30)$$

and $E(s) = s+c+\psi_1\alpha-(1-c)A(s)-\psi_2\left[B(s)+\psi_1 A(s)D_2(s+\psi_1)\right]\overline{S}_1(s)$ …(31)

It is important to note here that

Sum of equations (23) through (28) = $\dfrac{1}{s}$ …(32)

ASYMPTOTIC BEHAVIOUR OF CONSIDERED SYSTEM:

Using final value theorem of Laplace transform, viz., $\lim_{t\to\infty} P(t) = \lim_{s\to 0} s\,\overline{P}(s) = P$ (say), provided the limit on left side exists, we obtain the following asymptotic behavior of considered system from equations (23) through (28):

$$P_{0,S} = \frac{1}{E'(0)} \quad \ldots(33)$$

$$P_{F,0} = \frac{B(0)}{E'(0)} \quad \ldots(34)$$

$$P_{SW} = \frac{(1-\alpha)B(0)}{\beta E'(0)} \quad \ldots(35)$$

$$P_{0,C} = \frac{A(0)}{E'(0)} \quad \ldots(36)$$

$$P_{F,F} = \frac{\psi_2 B(0)}{E'(0)} M_1 \quad \ldots(37)$$

$$P_{0,F} = \frac{\psi_2 A(0)}{E'(0)} D_2(\psi_1) \quad \ldots(38)$$

where, $M_1 = -\overline{S}_1'(0) =$ Mean time to repair whole system

$$A(0) = \frac{c}{\psi_1\alpha+\psi_2\left[1-\overline{S}_2(\psi_1)\right]+(1-c)}$$

$$B(0)=\frac{\psi_1}{\psi_2}\alpha[1+A(0)]$$

and $E'(0)=\left[\frac{d}{ds}E(s)\right]_{s=0}$.

PARTICULAR CASE

(I) WHEN ALL REPAIRS FOLLOW EXPONENTIAL TIME DISTRIBUTION :

In this case, setting $\overline{S}_i(j)=r_i/(j+r_i)$, $\forall$ i and j, in equations (23) through (28), we obtain the following Laplace transforms of various states probabilities of fig-1:

$$\overline{P}_{0,S}(s)=\frac{1}{E_1(s)} \quad \ldots(39)$$

$$\overline{P}_{F,0}(s)=\frac{B_1(s)}{E_1(s)} \quad \ldots(40)$$

$$\overline{P}_{SW}(s)=\frac{(1-\alpha)B_1(s)}{(s+\beta)E_1(s)} \quad \ldots(41)$$

$$\overline{P}_{0,C}(s)=\frac{A_1(s)}{E_1(s)} \quad \ldots(42)$$

$$\overline{P}_{F,F}(s)=\frac{\psi_2 B_1(s)}{E_1(s)(s+r_1)} \quad \ldots(43)$$

$$\overline{P}_{0,F}(s)=\frac{\psi_2 A_1(s)}{E_1(s)(s+\psi_1+r_2)} \quad \ldots(44)$$

where, $$A_1(s)=\frac{c}{s+\psi_1\alpha+\frac{\psi_2(s+\psi_1)}{s+\psi_1+r_2}+(1-c)} \quad \ldots(45)$$

$$B_1(s)=\frac{\psi_1\alpha[1+A_1(s)]}{s+\psi_2+\frac{s(1-\alpha)}{s+\beta}} \quad \ldots(46)$$

and $$E_1(s)=s+c+\psi_1\alpha-\psi_2\left[B_1(s)+\frac{\psi_1 A_1(s)}{s+\psi_1+r_2}\right]\frac{r_1}{s+r_1}-(1-c)A_1(s) \quad \ldots(47)$$

(II) WHEN SWITCHING DEVICE USED IS PERFECT:

In this case, put $\alpha=1$ and $P_{SW}(t)=0$ in equations (23) through (28), we obtain required results.

AVAILABILITY OF CONSIDERED SYSTEM:

From equations (23) and (24), we have

$$\overline{P}_{up}(s)=\frac{1}{s+c+\psi_1\alpha}\left\{1+\frac{\psi_1\alpha}{s+\psi_2+(1-\alpha)}\left[1+\frac{c}{s+\psi_1\alpha+\psi_2}\right]\right\}$$

Taking inverse Laplace transform, we obtain

$$P_{up}(t)=\left\{1+\frac{\psi_1\alpha}{\psi_2+(1-\alpha)-c-\psi_1\alpha}\left[1+\frac{c}{\psi_2-c}\right]\right\}e^{-(c+\psi_1\alpha)t}$$

$$+\left\{\frac{\psi_1\alpha}{\psi_2+(1-\alpha)-c-\psi_1\alpha}\left[\frac{c}{(1-\alpha)-\psi_1\alpha}-1\right]\right\}e^{-[\psi_2+(1-\alpha)]t}$$

$$-\frac{\psi_1\alpha c}{[(1-\alpha)-\psi_1\alpha](\psi_2-c)}e^{-(\psi_1\alpha+\psi_2)t} \quad \ldots(48)$$

Also, $P_{down}(t)=1-P_{up}(t)$ …(49)

It is worth noticing that $P_{up}(0)=1$.

COST FUNCTION FOR CONSIDERED SYSTEM:

Cost function for considered system is given by

$$G(t)=C_1\int_0^t P_{up}(t)dt-C_2t \quad \ldots(50)$$

where, C_1 and C_2 are revenue and repair costs per unit time, respectively. Also,

$$\int_0^t P_{up}(t)dt=\frac{1}{c+\psi_1\alpha}\left\{1+\frac{\psi_1\alpha}{\psi_2+(1-\alpha)-c-\psi_1\alpha}\left[1+\frac{c}{\psi_2-c}\right]\right\}\left[1-e^{-(c+\psi_1\alpha)t}\right]$$

$$+\frac{1}{\psi_2+(1-\alpha)}\left\{\frac{\psi_1\alpha}{\psi_2+(1-\alpha)-c-\psi_1\alpha}\left[\frac{c}{(1-\alpha)-\psi_1\alpha}-1\right]\right\}\left[1-e^{-(\psi_2+1-\alpha)t}\right]$$

$$-\frac{\psi_1\alpha c}{(1-\alpha-\psi_1\alpha)(\psi_2-c)(\psi_1\alpha+\psi_2)}\left[1-e^{-(\psi_1\alpha+\psi_2)t}\right] \quad \ldots(51)$$

NUMERICAL ILLUSTRATION:

For a numerical illustration of obtained results, let us consider the following values: $\psi_1=0.06$, $\psi_2=0.08$, $\alpha=0.6$, $c=0.03$, $C_1=\text{Rs }7.00$, $C_2=\text{Rs }3.00$ and $t=0,1,2----10$. Using these values in equations (48) and (50) we compute table-2 and 3, respectively. The corresponding graphs have been shown in fig-2 and 3, respectively.

Table-2

t	$P_{up}(t)$
0	1
1	0.9619
2	0.91785
3	0.87103
4	0.82356
5	0.77670
6	0.73122
7	0.68756
8	0.64594
9	0.60646
10	0.56913

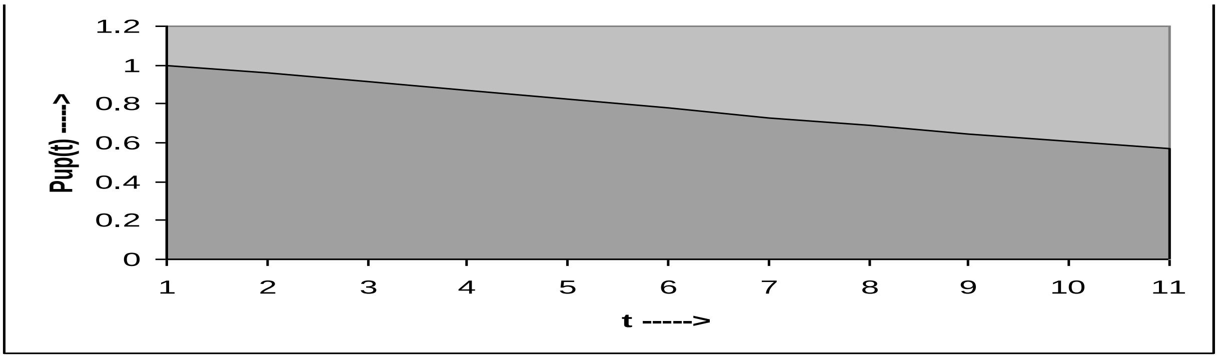

Fig-2: Availability Vs. Time

Table-3

t	$G(t)$		
	$G_1(t)$ $C_1 = 7,\ C_2 = 3$	$G_2(t)$ $C_1 = 10,\ C_2 = 6$	$G_3(t)$ $C_1 = 7,\ C_2 = 4$
0	0	0	0
1	4.87155	4.81650	3.87153
2	8.45319	8.21884	6.45393
3	10.7151	10.1645	8.71589
4	14.6461	13.6372	9.64678
5	17.2464	15.6372	11.2446
6	18.5232	17.1760	13.5223
7	20.4878	18.2683	14.4882
8	23.1538	18.9345	15.1584
9	24.5359	18.1945	15.5398
10	25.6493	18.0706	15.6432

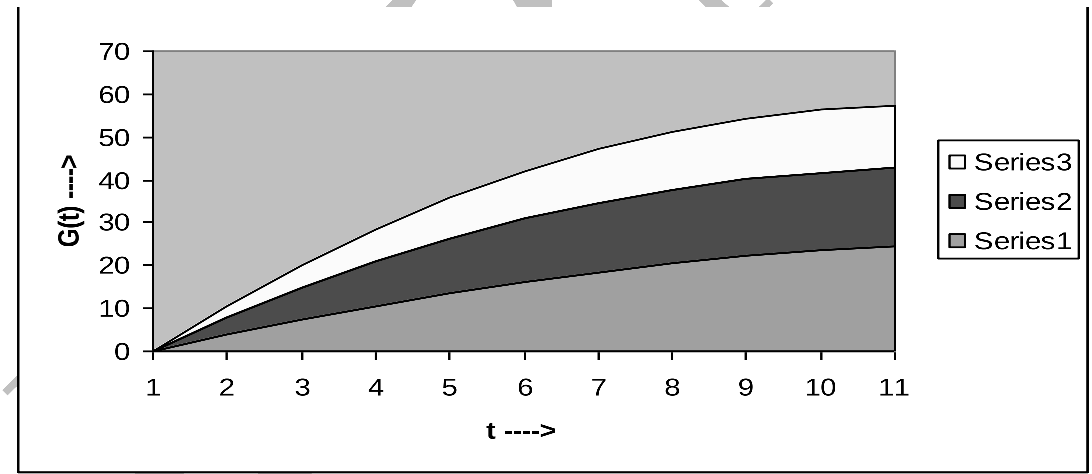

Fig-3 : Cost function versus time

RESULTS AND DISCUSSION:

Fig-2 shows the graph of 'Availability Vs time'. Analysis of fig-2 reveals that availability of considered system decreases approximately in constant manner with increase in time.

Fig-3 represents the graphs "Cost function Vs time". In this fig-3, we plot three cost functions $G_1(t), G_2(t)$ and $G_3(t)$ for different values of costs C_1 and C_2. We conclude that the values of cost function $G_1(t)$ remains better as compared with $G_2(t)$ and $G_3(t)$.

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